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"Induced" Super-Symmetry Breaking with a Vanishing Vacuum Energy

Ashok Das and Sergio A. Pernice

Department of Physics and Astronomy,
University of Rochester,
Rochester, New York, 14627.

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Abstract

A new mechanism for symmetry breaking is proposed which naturally avoids the constraints following from the usual theorems of symmetry breaking. In the context of super-symmetry, for example, the breaking may be consistent with a vanishing vacuum energy. A 2+1 dimensional super-symmetric gauge field theory is explicitly shown to break super-symmetry through this mechanism while maintaining a zero vacuum energy. This mechanism may provide a solution to two long standing problems, namely, dynamical super-symmetry breaking and the cosmological constant problem.

1 Introduction

Dynamical supersymmetry breaking remains a fundamental problem in particle physics [1, 2]. As pointed out long ago [3], supersymmetry algebra implies that if supersymmetry is unbroken, the vacuum must have a vanishing energy leading to an elegant "solution" of the cosmological constant problem [4]. But if supersymmetry is a symmetry of the physical theories, it must be broken, as is clear from the observed spectrum of particles. This, again from the supersymmetry algebra, would imply that the vacuum energy must be strictly positive - leading back to the cosmological constant problem. This is one of many situations where it would be desirable to break a symmetry bypassing the usual theorems on symmetry breaking. It is, therefore, worth investigating alternate scenarios where this might be achieved.

Let us recall some of the essential features of symmetry breaking [5, 6]. When a symmetry is broken, there is a set of degenerate vacuum states $|0_n>$. For simplicity, let us consider the breaking of a discrete symmetry with a finite number of such states which are assumed to be orthonormal

$$< 0_n|0_m >= \delta_{n,m} \tag{1}$$

The symmetry operation mixes these states and, therefore, they are degenerate. Every Hermitian local operator $A(x)$ of the theory will have only diagonal matrix elements

$$< 0_n|A(x)|0_m >= \delta_{n,m}a_m \tag{2}$$

The Hilbert spaces built on each of them are unitarily inequivalent. The distinct Hilbert spaces are, therefore, dynamically disconnected.

We see, then, that symmetry breaking has two essential ingredients: the existence of a symmetry of the theory, and the splitting of the Hilbert space into dynamically separated sectors. These two ingredients, however, need not always come together. The Hilbert space of a theory may split into dynamically disconnected parts *not* associated with an underlying symmetry (this, in the language of statistical mechanics, just amounts to loss of ergodicity, which is not necessarily associated with an underlying symmetry). The usual theorems on symmetry breaking [5], assume that the dynamically disconnected Hilbert spaces are mapped into each other by the symmetry operation. This is what actually happens in many field theories but it need not always be the case. For example, suppose our theory has *two* symmetries realized in the *full* Hilbert space and one is broken by the usual mechanisms. In this case, the states of the Hilbert space built on one of the degenerate vacua will carry representations of the broken (residual) symmetry and, unless there are specific relations between the two symmetries, may not contain all the states corresponding to the representations of the second symmetry. In the process, the multiplets of the second symmetry may get distributed into the dynamically disconnected Hilbert spaces. This would, of course, "induce" a breaking of the second symmetry in the physical sector of the Hilbert space and for such a breaking the usual theorems will no longer apply. Thus, for example, if because of spontaneous breaking of some symmetry in a theory, some of the superpartner states are removed from the physical Hilbert space, supersymmetry will be broken. This, however, would no longer necessarily imply positivity of the vacuum energy as we will see. Such a scenario would have the added advantage that some superpartner states would be completely unobservable (they do not belong to the physical Hilbert space) consistent with experimental observations. Furthermore, if such a phenomenon were to occur at low energies, the beautiful short distance properties following from supersymmetry would remain intact.

2 Dynamical Supersymmetry Breaking

In this letter we would like to explore the possible applications of these general notions. In particular, we will study a supersymmetric field theory where supersymmetry is explicitly shown to be

broken while the vacuum energy remains zero. Consider $2+1$ dimensional $N=2$ super-symmetric Abelian Chern-Simons theory given by the action

$$
\begin{aligned}
S &= \int \mathrm{d}^3x \left\{ \frac{\kappa}{4c}\, \varepsilon^{\mu\nu\lambda} A_\mu F_{\nu\lambda} + |D_\mu\phi|^2 + i\, \overline{\psi}\, \gamma^\mu D_\mu \psi \right. \\
&\qquad \left. - (\frac{e^2}{\kappa c^2})^2 |\phi|^2 \left[|\phi|^2 - v^2 \right]^2 + \frac{e^2}{\kappa c^2} \left[3|\phi|^2 - v^2 \right]\ \overline{\psi}\psi \right\}
\end{aligned} \tag{3}
$$

where $D_\mu = \partial_\mu + i(e/c)A_\mu$, ϕ is a complex spin-0 field, ψ is a complex two-component Dirac field corresponding to spin 1/2 particles, and κ may be positive or negative. We consider the positive case. There are therefore two degrees of freedom corresponding to the complex scalar field and two degrees of freedom corresponding to the fermion and its anti-particle (in 2+1 dimensions both the fermion and its anti-particle have only one spin degree of freedom). The Chern-Simons field A_μ carries no dynamical degrees of freedom in the symmetric phase so the super-symmetric counting of degrees of freedom follows. The mass of the supersymmetric partner particles is $m = e^2 v^2 / c^2 \kappa$. The model of eq. (3) was first studied by Lee, Lee and Weinberg [8].

In Ref. [9], Leblanc, Lozano and Min studied the symmetries of the low energy effective theory corresponding to (3). We review some of their results relevant for our work. In the low energy limit, (3) corresponds to a Galilean invariant $N=2$ supersymmetric field theory. In the zero antiparticle sector (in the nonrelativistic limit particle and antiparticle numbers are separately conserved) its Hamiltonian is given by

$$
H = \int \mathrm{d}^2 r \, \left\{ \frac{1}{2m} \left[(\mathbf{D}\Phi)^\dagger \cdot \mathbf{D}\Phi + (\mathbf{D}\Psi)^\dagger \, \mathbf{D}\Psi \right] - \frac{e}{2mc} : B\ \rho_F : - \frac{e^2}{2mc\kappa} : \rho_B^2 : -3 \frac{e^2}{2mc\kappa} \rho_b \rho_F \right\} \tag{4}
$$

where

$$
\rho_B = |\Phi|^2 \quad \text{and} \quad \rho_F = |\Psi|^2 \tag{5}
$$

and the (scalar) magnetic field is given by $B = \nabla \times \mathbf{A}$[1]. The bosonic field Φ corresponds to the nonrelativistic limit, in the zero antiparticle sector, of the original field ϕ.The fermionic field Ψ corresponds also to the nonrelativistic limit, in the zero antiparticle sector, of the first component of the of the two component original fermionic field ψ. The second component has been eliminated through its equation of motion. In the covariant derivatives the gauge field is replaced through its equation of motion in Coulomb gauge [9, 10, 11] by

$$
\mathbf{A}(t, \mathbf{r}) = \frac{e}{\kappa} \nabla \times \int \mathrm{d}^2 r' G(\mathbf{r}' - \mathbf{r})\ \rho(t, \mathbf{r}') \tag{6}
$$

where $\rho = \rho_B + \rho_F$ and $G(\mathbf{r})$ is the two dimensional Green's function of the Laplacian

$$
G(\mathbf{r}) = \frac{1}{2\pi} \ln |\mathbf{r}| \tag{7}
$$

The model defined by the Hamiltonian (4) corresponds to the super-symmetric extension of a model studied by Jackiw and Pi [11]. We see then from (4) that the nonrelativistic limit of (3)

[1] In two dimensions, the cross product of two vectors $\mathbf{V} \times \mathbf{W} = \varepsilon^{ij} V_i W_j$ is a scalar, the curl of a vector (also a scalar) is $\nabla \times \mathbf{V} = \varepsilon^{ij} \partial_i V^j$, and the curl of a scalar is a vector with components $(\nabla \times S)^i = \varepsilon^{ij} \partial_j S$.

corresponds to bosons interacting (minimally) with the gauge field and with themselves through a contact interaction. The fermions also interact minimally with the gauge fields but in addition they posses a Pauli interaction. This non-minimal interaction arose algebraically from the elimination of the second component of the spinor field through its equation of motion, but its appearance can also be expected, on general grounds, in the nonrelativistic limit of super-symmetric gauge theories [12]. Finally, there is a contact boson-fermion interaction.

In [9] it was shown that the model (4) has 16 generators of symmetry operations that generate an extended super-conformal Galilean algebra. Of immediate interest to us is the algebra of the generators of supersymmetry. It corresponds to a $N = 2$ Galilean supersymmetry. The two supercharges in the nonrelativistic theory are given respectively by[2]

$$Q_1 = i\sqrt{2m}\int \mathrm{d}^2 r\ \Phi^\dagger \Psi \tag{8}$$

$$Q_2 = i\frac{1}{\sqrt{2m}}\int \mathrm{d}^2 r\ \Phi^\dagger D_+ \Psi \tag{9}$$

and together with the Hamiltonian (4) they satisfy the super symmetry algebra

$$\left\{Q_1, Q_1^\dagger\right\} = 2M \tag{10}$$

$$\left\{Q_2, Q_2^\dagger\right\} = H \tag{11}$$

$$\left\{Q_1, Q_2^\dagger\right\} = P_- \tag{12}$$

$$\{Q_\alpha, Q_\beta\} = \left\{Q_\alpha^\dagger, Q_\beta^\dagger\right\} = 0 \tag{13}$$

The mass operator in eq. (10), which commutes with all other generators, is necessary in a Galilean invariant theory to close the algebra [7].

According to the Bargmann superselection rule [7], a nonrelativistic supersymmetric field theory should be equivalent to a supersymmetric Schrödinger equation in each particle number sector of the theory. We will analyze in particular the two particle sector, that is both tractable and non-trivial, which can be derived from the above expressions assuming the existence of a zero particle, zero energy vacuum. Following ref. [9], let us define the orthonormal two particle states $|E, N_B, N_F>$ with energy E, N_B number of bosons and N_F number of fermions as follows

$$|E,2,0> = \frac{1}{\sqrt{2}}\int \mathrm{d}r_1 \mathrm{d}r_2\ u_B(\mathbf{r_1},\mathbf{r_2})\Phi^\dagger(\mathbf{r_1})\Phi^\dagger(\mathbf{r_2})|\Omega> \tag{14}$$

$$|E,0,2> = \frac{1}{\sqrt{2}}\int \mathrm{d}r_1 \mathrm{d}r_2\ u_F(\mathbf{r_1},\mathbf{r_2})\Psi^\dagger(\mathbf{r_1})\Psi^\dagger(\mathbf{r_2})|\Omega> \tag{15}$$

$$\begin{aligned} |E,1,1> &= \frac{1}{\sqrt{2}}|E,1,1>_S + \frac{1}{\sqrt{2}}|E,1,1>_A \\ &= \frac{1}{2}\int \mathrm{d}r_1 \mathrm{d}r_2\ \Big\{u_S(\mathbf{r_1},\mathbf{r_2})\left[\Phi^\dagger(\mathbf{r_1})\Psi^\dagger(\mathbf{r_2}) + \Phi^\dagger(\mathbf{r_2})\Psi^\dagger(\mathbf{r_1})\right]|\Omega> \\ &\quad + u_A(\mathbf{r_1},\mathbf{r_2})\left[\Phi^\dagger(\mathbf{r_1})\Psi^\dagger(\mathbf{r_2}) - \Phi^\dagger(\mathbf{r_2})\Psi^\dagger(\mathbf{r_1})\right]|\Omega>\Big\} \end{aligned} \tag{16}$$

[2] For any vector $\mathbf{V} = (V^1, V^2)$, $V^\pm = V^1 \pm iV^2$.

The one fermion, one boson state is then divided into its symmetric and its antisymmetric part under the interchange of the two particles.

In the basis

$$\Upsilon = \begin{pmatrix} u_B \\ u_S \\ u_A \\ u_F \end{pmatrix} \tag{17}$$

the supercharges (8) and (9) have the form

$$Q_1 = 2i\sqrt{m} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad , \quad Q_2 = \frac{1}{2\sqrt{m}} \begin{pmatrix} 0 & \mathcal{D}_1^+ + \mathcal{D}_2^+ & -\mathcal{D}_1^+ + \mathcal{D}_2^+ & 0 \\ 0 & 0 & 0 & \mathcal{D}_1^+ - \mathcal{D}_2^+ \\ 0 & 0 & 0 & \mathcal{D}_1^+ + \mathcal{D}_2^+ \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{18}$$

where

$$\mathcal{D}_1 = \nabla_1 - \frac{ie^2}{c\kappa}\nabla_1 \times G(\mathbf{r_1} - \mathbf{r_2}) \quad , \quad \mathcal{D}_2 = \nabla_2 - \frac{ie^2}{c\kappa}\nabla_2 \times G(\mathbf{r_2} - \mathbf{r_1}) \tag{19}$$

The two particle Hamiltonian, in the same basis, is

$$H = \left\{ -\frac{1}{2m}\left(\mathcal{D}_1^2 + \mathcal{D}_2^2\right) I - \frac{e^2}{mc\kappa}\, \delta(\mathbf{r_1} - \mathbf{r_2}) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \right\} \tag{20}$$

From eqs. (18) and (20) one can easily check that the algebra of eqs. (10-13) is satisfied with the mass operator corresponding to $M = 2mI$ as it should be in the two particle sector.

From eq. (18) and (19) it is clear that the 12 and the 34 components of Q_2 refer to the center of mass momentum and have no dynamical effect. In order to investigate the dynamical content of the system under consideration we will, therefore, work in the center of mass reference frame where they vanish. In that reference frame, in terms of the relative variable $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ and in polar coordinates, the only nonvanishing components of Q_2 have the form

$$Q_2^{13} = -Q_2^{24} = -\frac{e^{i\theta}}{\sqrt{m}}\left[\frac{\partial}{\partial r} + \frac{1}{r}i\frac{\partial}{\partial\theta} - \frac{e^2}{c\kappa}\frac{\partial G}{\partial r}\right] \tag{21}$$

Similarly, for $Q_2^\dagger$ the only nonvanishing components are (remember that in two dimensions $(\partial/\partial r)^\dagger = -\partial/\partial r - 1/r$)

$$\left(Q_2^\dagger\right)^{31} = -\left(Q_2^\dagger\right)^{42} = -\frac{1}{\sqrt{m}}\left[-\frac{\partial}{\partial r} + \frac{1}{r}\left(i\frac{\partial}{\partial\theta} - 1\right) - \frac{e^2}{c\kappa}\frac{\partial G}{\partial r}\right] e^{-i\theta} \tag{22}$$

From eqs. (21) and (22), remembering that these charges are defined in the basis given in eq. (17), we find the following relations between superpartner states: the superpartner of a boson-boson

state with orbital angular momentum 2ℓ, $|bb, 2\ell>$ (i.e., the state, $Q_2^\dagger\ |bb, 2\ell>$), is a boson-fermion antisymmetric state with orbital angular momentum $2\ell+1$, that is, $|bf_A, 2\ell+1>$. A similar relation holds for a boson-fermion symmetric state $|bf_S, 2\ell>$ and its fermion-fermion superpartner. The difference in one unit of orbital angular momentum for superpartner states (which is perfectly compatible with $\Delta J = 1/2$ in relativistic field theories) was shown to be a general property of two dimensional supersymmetric quantum mechanics in ref. [12]. This property holds for the supercharge related to the Hamiltonian in the supersymmetry algebra (Q_2 in our case). In terms of the general formalism of ref. [12], the interaction part of the supercharges corresponds to a vector superpotential $\vec{W} = i(e^2/c\kappa)(\partial G/\partial r)\ \hat{\theta}$. $\hat{\theta}$ refers to the unit vector in the angular direction.

We will now study the eigenvalue equations for the superpartners. The analysis for the super pair boson-boson and boson-fermion antisymmetric is identical to the other super pair (boson-fermion symmetric and fermion-fermion), so we only analyze the former. The eigenvalue equation for the state $u_B(r)\ e^{-i2\ell\ \theta} =< \mathbf{r}|\ bb, 2\ell>$ is

$$Q_2^{13}\left(Q_2^\dagger\right)^{31} u_B(r) = \frac{1}{m}\left[-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{(2\ell-\lambda)^2}{r^2} - \lambda\frac{\delta(r)}{r}\right] u_B = E u_B(r) \qquad (23)$$

where $\lambda = e^2/(2\pi c\kappa)$. On the other hand, the superpartner state

$$u_A(r)\ e^{-i(2\ell+1)\ \theta} =< \mathbf{r}|\ \left(Q_2^\dagger\right)^{31} |bb, 2\ell> =< \mathbf{r}|\ bf_A, 2\ell+1> \qquad (24)$$

satisfies the equation

$$\left(Q_2^\dagger\right)^{31} Q_2^{13}\ u_A(r) = \frac{1}{m}\left[-\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{(2\ell+1-\lambda)^2}{r^2} + \lambda\frac{\delta(r)}{r}\right] u_A = E u_A(r) \qquad (25)$$

As we see, the boson-boson state feels an attractive delta function interaction while the boson-fermion antisymmetric state a repulsive one. In this second case, the delta function might be omitted since, an antisymmetric u_A would make it vanish at the origin. On the other hand, u_A is the superpartner of u_B, so it must be generated from it through the action of $Q_2^\dagger$. It is therefore more natural to leave it in the equations. As we will see shortly, the sign of the delta function interaction does not have any relevance. Whether attractive or repulsive, the dynamical effect of it is to suppress non-regular solutions (i.e., solutions not vanishing at the origin). Therefore everything is consistent, and as we will see, a great surprise results from this consistency. Equations (23) and (25) clearly show that, apart from the delta function interactions, the dynamics of this model corresponds to a supersymmetric Aharonov-Bohm problem [13]. This is not surprising since the (nonsupersymmetric) Galilean invariant Chern-Simons theory is a field theoretic representation of this problem [10].

In the Aharonov-Bohm context, the solutions of eqs. (23) and (25) are set to zero at the origin. This makes physical sense because the solenoid where the magnetic field is confined is suppose to be isolated from the particle by some other interaction (this interaction forces the wave function to vanish at the origin). The purpose of Aharonov and Bohm was precisely to show that in quantum mechanics the *potential* has a physical effect, even when the particle is never in contact with the

magnetic field. In our context, however, equations (23) and (25) were derived as the nonrelativistic limit of a relativistic supersymmetric field theory. This field theory is suppose to encode the totality of the interactions of the system represented by it. Therefore, we cannot force by hand the behavior of the wave functions at the origin - specially for the symmetric boson-boson case. However, as mentioned before, the delta function interaction dictates *dynamically* the behavior at the origin. It forces the solutions to vanish there, as is the case (by hand) in the Aharonov-Bohm problem. For the boson-fermion antisymmetric case, this is just a self consistent result. But for the boson-boson case it involves and independent dynamical effect. So, although the solutions of eqs. (23) and (25) are well known, it is worth re-deriving them here once more in a form that explicitly exposes the aspects relevant for our purpose.

As mentioned, the sign and the magnitude of the coefficient of the delta function turns out to be irrelevant (if different from zero). Therefore let us consider the equation

$$\left[-\frac{\partial^2}{\partial r^2} + \frac{\alpha^2 - 1/4}{r^2} + \beta \frac{\delta(r-R)}{R} - E \right] v = 0 \tag{26}$$

that includes all possible cases with the identification $v = u\sqrt{r}$ and $\alpha = 2\ell - \lambda$, $\beta = -\lambda$ for the boson-boson case and $\alpha = 2\ell + 1 - \lambda$, $\beta = \lambda$ for the boson-fermion antisymmetric case. The delta function has been regularized; at the end of the calculations we will take the $R \to 0$ limit. The results should be independent of the particular regularization chosen [14].

Choosing, for $r \neq R$, a solution of the form

$$v = \sum_{n=0}^{\infty} a_n r^{n+\delta} \tag{27}$$

one finds $\delta = 1/2 \pm |\alpha|$. While for the plus sign the corresponding solution $u = v/\sqrt{r}$ vanishes at the origin, for the minus sign it diverges there. However, note that if $|\alpha| < 1$ the minus sign solution is still normalizable. Without making any assumption about the behavior at the origin we have

$$v(r) = A \sum_{n=0}^{\infty} a_{2n}^{+}\ r^{2n+1/2+|\alpha|} + B \sum_{n=0}^{\infty} a_{2n}^{-}\ r^{2n+1/2-|\alpha|} \quad , \qquad \text{for} \quad r > R \tag{28}$$

$$v(r) = C \sum_{n=0}^{\infty} a_{2n}^{+}\ r^{2n+1/2+|\alpha|} + D \sum_{n=0}^{\infty} a_{2n}^{-}\ r^{2n+1/2-|\alpha|} \quad , \qquad \text{for} \quad r < R \tag{29}$$

where the coefficients a_{2n}^{+} and a_{2n}^{-} satisfy the relation

$$a_{2n+2}^{+} = \frac{-E}{(2n+2+2|\alpha|)\,(2n+2)} a_{2n}^{+} \tag{30}$$

$$a_{2n+2}^{-} = \frac{-E}{(2n+2-2|\alpha|)\,(2n+2)} a_{2n}^{-} \tag{31}$$

We see from these equations that the energy has to be positive or zero. If it is negative the sign of the coefficients would not oscillate and the wave functions would grow exponentially for large r.

At $r = R$, which we assume to be arbitrarily small (remember that the limit $R \to 0$ is supposed to be taken at the end of the calculation), the following two equations have to be satisfied

$$v(R+\epsilon) = v(R-\epsilon) \tag{32}$$

$$v'(R-\epsilon) - v'(R+\epsilon) = \frac{\beta}{R} v(R) \tag{33}$$

Let us, for simplicity of analysis, assume that $|\alpha| < 1$.The continuity equation implies $(A-C)R^{2|\alpha|} = (D-B)$. In the limit $R \to 0$ this leads to $B = D$, with no constraints over A or C. Equation (33), together with the above result, implies $[(1/2 + |\alpha|)(C - A) + \beta C]\, R^{2|\alpha|} + \beta D = 0$. In the limit $R \to 0$ it imposes $\beta D = 0$. If $\beta \neq 0$, independent of its sign, this implies $D = 0$, and, therefore, $B = 0$ as well. We can easily show that this conclusion holds for any $|\alpha|$.

We see then that the *dynamics* determine the behavior at the origin. For $|\alpha| > 1$, the irregular solutions could have been discarded invoking normalizability. It is nice that the dynamics gets rid of such solutions. For $|\alpha| < 1$, normalizability alone is not enough to discard the irregular solutions. Even more, the divergence of the wave function at the origin could have been understood as the effect of the delta function interaction in the attractive case (corresponding to the boson-boson states). However, as we just saw, whether attractive or repulsive, the effect of the delta function interaction is to force the irregular solutions out. This is also consistent with the antisymmetric nature of the boson-fermion states. So we conclude that in the general case, only solutions vanishing at the origin survive.

Remember that equation (26) represents (in the reduced variables) equation (23) for a boson-boson state of orbital angular momentum 2ℓ with the identification $\alpha = 2\ell - \lambda$. It also represents equation (25) for the boson-fermion antisymmetric superpartner state of orbital angular momentum $2\ell + 1$ with the identification $\alpha = 2\ell + 1 - \lambda$. For our purposes it is convenient to consider both regular and irregular solutions, keeping in mind that the irregular ones do not belong to the physical Hilbert space. For definiteness, let us assume

$$0 < \lambda = \frac{e^2}{2\pi c\kappa} < 1 \tag{34}$$

Consider the regular solutions corresponding to the boson-boson state of orbital angular momentum 2ℓ for $\ell = 0, -1, -2, \cdots$. Independent of their energy, at the origin, these solutions go to zero as $r^{-(2\ell-\lambda)}$. The superpartner solutions, that is, the solutions obtained by applying $\left(Q_2^\dagger\right)^{31}$ to them, have orbital angular momentum $2\ell + 1$ and behave near the origin as $r^{-(2\ell+1-\lambda)}$. For $\ell = 0$, again independent of their energy, this corresponds to an irregular (but normalizable) solution that, as we have shown, does not belong to the physical Hilbert space. Therefore, the boson-boson states of *arbitrary energy* with zero orbital angular momentum do not have a superpartner! Similarly, consider now the irregular solutions (that do not belong to the physical Hilbert space) with arbitrary energy and negative or zero orbital angular momentum. Near the origin they behave as $r^{(2\ell-\lambda)}$. Note that for $\ell = 0$, although not in the physical Hilbert space, the solution is normalizable. Applying $\left(Q_2^\dagger\right)^{31}$ to them, we obtain solutions of the superpartner equation that near the origin behave as $r^{(2\ell+1-\lambda)}$. For $\ell = 0$ the exponent is positive, therefore, it corresponds to a regular

solution that belongs to the physical Hilbert space. We conclude then that the boson-fermion antisymmetric states of orbital angular momentum equal to one and arbitrary energy do not have a superpartner! Furthermore, the zero orbital angular momentum boson-boson solutions and the boson-fermion antisymmetric solutions of orbital angular momentum equal to one that do belong to the physical Hilbert space are *not* superpartners of each other. The dynamical separation of the physical Hilbert space produced by the delta function interaction, among other things, pushes the irregular (but normalizable) solutions outside the physical Hilbert space, and induced the breaking of supersymmetry!

Let us show explicitly the above statements. From eqs. (27), (30) and (31), remembering that $\delta = 1/2 \pm |\alpha|$, that $u = v/\sqrt{r}$, and choosing for convenience $a_0^{\pm} = \sqrt{E}^{\ \pm|\alpha|}/(\pm 2|\alpha|)$, we obtain for the regular $(+)$ and the irregular $(-)$ solutions, the expressions

$$u^{\pm}(r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!!(2n \pm 2|\alpha|)!!} \left(\sqrt{E}r\right)^{2n\pm|\alpha|} \tag{35}$$

where $(2n \pm 2|\alpha|)!! = (2n \pm 2|\alpha|)(2n - 2 \pm 2|\alpha|) \cdots (\pm 2|\alpha|)$ and similarly for $(2n)!!$. Therefore, for a boson-boson state with zero or negative orbital angular momentum (zero or negative ℓ), the regular and irregular solutions are, respectively,

$$u_B^{+}(\mathbf{r}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!!(2n - 2\,(2\ell - \lambda))!!} \left(\sqrt{E}r\right)^{2n-(2\ell-\lambda)} e^{-i2\ell\theta} \tag{36}$$

$$u_B^{-}(\mathbf{r}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!!(2n + 2\,(2\ell - \lambda))!!} \left(\sqrt{E}r\right)^{2n+(2\ell-\lambda)} e^{-i2\ell\theta} \tag{37}$$

When we act on them with $\left(Q_2^{\dagger}\right)^{31}$, given in equation (22), we obtain

$$\left(Q_2^{\dagger}\right)^{31} u_B^{+}(\mathbf{r}) \propto \sum_{n=0}^{\infty} \frac{(-1)^n(2n - 2\,(2\ell - \lambda))}{(2n)!!(2n - 2\,(2\ell - \lambda))!!} \left(\sqrt{E}r\right)^{2n-(2\ell-\lambda)-1} e^{-i(2\ell+1)\theta} \tag{38}$$

$$\left(Q_2^{\dagger}\right)^{31} u_B^{-}(\mathbf{r}) \propto \sum_{n=0}^{\infty} \frac{(-1)^n\ 2n}{(2n)!!(2n + 2\,(2\ell - \lambda))!!} \left(\sqrt{E}r\right)^{2n+(2\ell-\lambda)-1} e^{-i(2\ell+1)\theta} \tag{39}$$

The regular solution then trivially can be written in the form

$$\left(Q_2^{\dagger}\right)^{31} u_B^{+}(\mathbf{r}) \propto \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!!(2n - 2\,(2\ell + 1 - \lambda))!!} \left(\sqrt{E}r\right)^{2n-(2\ell+1-\lambda)} e^{-i(2\ell+1)\theta} \tag{40}$$

which corresponds to the regular solutions of orbital angular momentum $2\ell + 1$ except for $\ell = 0$. In that case the solution becomes irregular and does not belong to the Hilbert space as mentioned above.

For eq. (39) the situation is only slightly more involved. Being the coefficients proportional to $2n$, the $n = 0$ one vanishes while in the others $2n/(2n)!! = 1/(2(n-1))!!$. Shifting variables with

$j = n - 1$ and calling again n to j we obtain

$$\left(Q_2^\dagger\right)^{31} u_B^-(\mathbf{r}) \propto \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!!(2n + 2\,(2\ell + 1 - \lambda))!!} \left(\sqrt{E}r\right)^{2n+(2\ell+1-\lambda)} e^{-i(2\ell+1)\theta} \tag{41}$$

They correspond to irregular solutions with orbital angular momentum equal to $2\ell + 1$ (remember that ℓ is negative or zero) except for $\ell = 0$. In that case, eq. (41) corresponds to a regular solution and therefore belongs to the physical Hilbert space. This finishes the proof of the breaking of supersymmetry.

3 Discussion

So, as we have seen, the whole tower of boson-boson states of zero orbital angular momentum and arbitrary energy do not have superpartners. Similarly, the entire tower of boson-fermion antisymmetric states of orbital angular momentum equal to one and arbitrary energy do not have superpartners. Clearly the same is true for the other supersymmetric pair as well. The supersymmetry, Q_2, of the Hamiltonian is broken, but this does not imply a Goldstino in the spectrum. Note that in the context of a nonabelian gauge interaction, where we expect a confining superpotential, the states without superpartners (corresponding to lowest angular momentum) would naturally correspond to the lower energy bound states. Therefore we would expect a spectrum consisting of low energy bound states without superpartners. This is, of course, what is to be expected on physical grounds.

In order to derive the equations in the two particle sector, the existence of a zero energy vacuum state was assumed. The situation in the two particle sector, where the ground state is clearly a zero (nonrelativistic) energy state, is illuminating in this respect. In this sector we bypass the positivity of the ground state energy (that "follows" from the nonrelativistic supersymmetry algebra) under broken supersymmetry because, as we have just seen, the breaking is of a very special nature. Here, some of the superpartner states do not belong to the physical Hilbert space. They are not in the domain of the supercharge operators (a similar observation was noted in connection with one dimensional supersymmetric quantum mechanics in ref. [15]). A vanishing ground state energy is therefore perfectly consistent.

It is important to understand the underlying reasons for the "induced" supersymmetry breaking in order to look for realistic theories where similar mechanisms might be operating. In this connection, let us note that the model (3), is not invariant under parity. Another way of saying the same is that a parity operation leads to a different theory. From eqs. (36-37) we note that for $l = 0$ (the supersymmetry breaking states), when λ goes to $-\lambda$ (which is reminiscent of the parity transformation), the regular solution goes into the irregular one. So the induced breaking of supersymmetry is very likely related to the parity violation in this theory. This issue, however, deserves a much more careful and detailed investigation which we leave for another paper. For the

moment let us simply point out that in nature parity is not conserved. It is suggestive and would, therefore, be interesting if parity (or an alternate discrete symmetry such as CP) violation can be tied to "induced" supersymmetry breaking in relativistic $3+1$ dimensional theories. This would not only explain why superpartner particles have been so elusive but also would lead to a solution of the cosmological constant problem. At present, we are trying to find a realistic model which would incorporate "induced" supersymmetry breaking.

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